

BAB 1

OPERASI ALJABAR

Beberapa Sifat operasi pada bilangan

1. $ab = ba$
2. $(ab)c = a(bc) = abc$
3. $a(b+c) = ab+ac$
4. $a(b-c) = ab-ac$
5. $(a+b)(c+d) = ac+ad+bc+bd$

Contoh 1: $(4x + 1)(3x - 2) = \dots$

Jawab

$$\begin{aligned} (4x + 1)(3x - 2) &= (4x)(3x) + (4x)(-2) + 1(3x) + 1(-2) \\ &= 12x^2 - 8x + 3x - 2 \\ &= 12x^2 - 5x - 2 \end{aligned}$$

Contoh 2: $(2x - 5)(2x - 7) = \dots$

Jawab

$$\begin{aligned} (2x - 5)(2x - 7) &= (2x)(2x) + (2x)(-7) + (-5)(2x) + (-5)(-7) \\ &= 4x^2 - 14x - 10x + 35 \\ &= 4x^2 - 24x + 35 \end{aligned}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Ingat: $(a+b)^n \neq a^n + b^n$

$$(a-b)^n \neq a^n - b^n$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$(a \pm b)^4 = a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4$$

Contoh 3: $(2x + 3y)^2 =$

$$\begin{aligned} &= (2x)^2 + 2 \cdot 2x \cdot 3y + (3y)^2 \\ &= 4x^2 + 12xy + 9y^2 \end{aligned}$$

Contoh 4: $(5x - 8y)^2 =$

Jawab

$$\begin{aligned} (5x - 8y)^2 &= (5x)^2 - 2 \cdot 5x \cdot 8y + (8y)^2 \\ &= 25x^2 - 80xy + 64y^2 \end{aligned}$$

Contoh 5: $(2a + 3b)^3 =$

Jawab

$$\begin{aligned} (2a + 3b)^3 &= 1(2a)^3 + 3(2a)^2(3b) + 3(2a)(3b)^2 + 1(3b)^3 \\ &= 8a^3 + 36a^2b + 54ab^2 + 27b^3 \end{aligned}$$

Contoh 6: $(3p - 2q)^3 =$

Jawab

$$\begin{aligned} (3p - 2q)^3 &= 1(3p)^3 - 3(3p)^2(2q) + 3(3p)(2q)^2 - 1(2q)^3 \\ &= 27p^3 - 54p^2q + 36pq^2 - 8q^3 \end{aligned}$$

Contoh 7: $(a + 2b)^4 =$

Jawab

$$\begin{aligned} (a + 2b)^4 &= a^4 + 4a^3(2b) + 6a^2(2b)^2 + 4a(2b)^3 + 1(2b)^4 \\ &= a^4 + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4 \end{aligned}$$

Contoh 8: $(2a - 2b)^4 =$

Jawab

$$\begin{aligned} (2a + 3b)^4 &= 1(2a)^4 - 4(2a)^3(3b) + 6(2a)^2(3b)^2 \\ &\quad - 4(2a)(3b)^3 + 1(3b)^4 \\ &= 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4 \end{aligned}$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)$$

Contoh 9: $(2x - 3y - z)^2 =$

$$\begin{aligned} (2x - 3y - z)^2 &= (2x)^2 + (-3y)^2 + (-z)^2 + 2(-6xy - 2xz + 3yz) \\ &= 4x^2 + 9y^2 + z^2 - 12xy - 4xz + 6yz \end{aligned}$$

Memfaktorkan

$$ab + ac = a(b + c)$$

$$ab + ac = a(b - c)$$

$$\begin{aligned} ac + ad + bc + bd &= a(c + d) + b(c + d) \\ &= (a + b)(c + d) \end{aligned}$$

Contoh 10: $12x^2 + 32x =$

Jawab

$$\begin{aligned} 12x^2 + 32x &= (4x)(3x) + (4x)(8) \\ &= 4x(3x + 8) \end{aligned}$$

Contoh 11: $20a^5b^4 + 15a^2b^7 =$

Jawab

$$\begin{aligned} 20a^5b^4 + 15a^2b^7 &= (5a^2b^4)(4a^3) + (5a^2b^4)(3b^3) \\ &= 5a^2b^4(4a^3 + 3b^3) \end{aligned}$$

$$a^2 - b^2 = (a + b)(a - b)$$

Contoh 12: $x^2 - 16$

Jawab

$$\begin{aligned} x^2 - 16 &= x^2 - 4^2 \\ &= (x + 4)(x - 4) \end{aligned}$$

Contoh 13: $36x^2 - 81y^2 =$

Jawab

$$\begin{aligned} 36x^2 - 81y^2 &= (6x)^2 - (9y)^2 \\ &= (6x + 9y)(6x - 9y) \end{aligned}$$

$$\text{Jika } b = m + n$$

$$c = mn$$

$$\begin{aligned} x^2 + bx + c &= x^2 + (m + n)x + mn \\ &= (x + m)(x + n) \end{aligned}$$

Contoh 14: $x^2 + 11x + 24 =$

Jawab

$$\begin{aligned} x^2 + 11x + 24 &= x^2 + (8+3)x + 8 \cdot 3 \\ &= (x + 8)(x + 3) \end{aligned}$$

Contoh 15: $x^2 - 12x + 32 =$

Jawab

$$\begin{aligned} x^2 - 12x + 32 &= x^2 + ((-4) + (-8))x + (-4) \cdot (-8) \\ &= (x - 8)(x - 4) \end{aligned}$$

Contoh 16: $x^2 - x - 42 =$

$$\begin{aligned} x^2 - x - 42 &= x^2 + ((-8) + 7)x + (-8) \cdot 7 \\ &= (x - 8)(x + 7) \end{aligned}$$

$$\text{Jika } m + n = b$$

$$mn = ac$$

$$ax^2 + bx + c = \frac{1}{a}(ax + m)(ax + n)$$

Contoh 17: $2x^2 + 11x + 12 =$

Jawab

$$m + n = b = 11$$

$$mn = ac = 2 \cdot 12 = 24$$

Diperoleh $m = 8$ dan $n = 3$

$$\begin{aligned} 2x^2 + 11x + 12 &= \frac{1}{2}(2x + 8)(2x + 3) \\ &= (x + 4)(2x + 3) \end{aligned}$$

Contoh 18: $3x^2 - 2x - 16 =$

Jawab

$$m + n = b = -2$$

$$mn = ac = 3(-16) = -48$$

Diperoleh $m = -8$ dan $n = 6$

$$\begin{aligned} 3x^2 - 2x - 16 &= \frac{1}{3}(3x + 8)(3x - 6) \\ &= (3x + 8)(x - 2) \end{aligned}$$

Contoh 18: $6x^2 - 13x + 6 =$

Jawab

$$m + n = b = -13$$

$$mn = ac = 6 \cdot 6 = 36$$

Diperoleh $m = -9$ dan $n = -4$

$$\begin{aligned} 6x^2 - 13x + 6 &= \frac{1}{6}(6x - 9)(6x - 4) \\ &= \left(\frac{1}{3}(6x - 9)\right)\left(\frac{1}{2}(6x - 4)\right) \\ &= (2x - 3)(3x - 2) \end{aligned}$$

KAJI LATIH STANDAR 1

OPERASI ALJABAR

- $(2x - 5)(x + 4) = \dots$
 - $2x^2 + 3x - 20$
 - $2x^2 + 11x - 21$
 - $2x^2 - 21x - 11$
 - $2x^2 - 21x + 11$
 - $2x^2 + 11x + 21$
- $(3x + 7)(x - 2) = \dots$
 - $3x^2 + x - 14$
 - $3x^2 - x + 14$
 - $3x^2 + x + 14$
 - $3x^2 - x - 14$
 - $3x^2 + 13x - 14$
- $(4x + 7y)^2 = \dots$
 - $4x^2 + 14xy + 9y^2$
 - $4x^2 + 28xy + 49y^2$
 - $9x^2 + 56xy + 16y^2$
 - $16x^2 + 56xy + 49y^2$
 - $16x^2 + 28xy + 49y^2$
- $(4x - 7y)^2 = \dots$
 - $4x^2 - 14xy + 9y^2$
 - $4x^2 - 28xy + 49y^2$
 - $9x^2 - 56xy + 16y^2$
 - $16x^2 - 56xy + 49y^2$
 - $16x^2 - 28xy + 49y^2$
- $(5p + q)^3 = \dots$
 - $5p^3 + 15p^2q + 12pq^2 + 2q^3$
 - $5p^3 + 75p^2q + 6pq^2 + q^3$
 - $125p^3 + 15p^2q + 15pq^2 + q^3$
 - $125p^3 + 75p^2q + 15pq^2 + q^3$
 - $125p^3 - 75p^2q + 15pq^2 - q^3$
- $(2p - q)^3 = \dots$
 - $3p^3 - 18p^2q + 12pq^2 - 2q^3$
 - $4p^3 - 12p^2q + 6pq^2 - q^3$
 - $4p^3 - 6p^2q + 6pq^2 - q^3$
 - $8p^3 - 54p^2q + 36pq^2 - q^3$
 - $8p^3 - 12p^2q + 6pq^2 - q^3$
- $(a + 3b)^4 = \dots$
 - $a^4 + 12a^3b + 28a^2b^2 + 108ab^3 + 9b^4$
 - $a^4 + 12a^3b + 54a^2b^2 + 108ab^3 + 81b^4$
 - $a^4 + 96a^3b + 216a^2b^2 + 216ab^3 + 81b^4$
 - $a^4 + 12a^3b + 432a^2b^2 + 432ab^3 + 81b^4$
 - $81a^4 - 192a^3b + 432a^2b^2 + 432ab^3 + 324b^4$
- $(a - 3b)^4 = \dots$
 - $a^4 - 12a^3b + 28a^2b^2 - 108ab^3 + 9b^4$
 - $a^4 - 12a^3b + 54a^2b^2 - 108ab^3 + 81b^4$
 - $a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$
 - $a^4 - 12a^3b + 432a^2b^2 - 432ab^3 + 81b^4$
 - $81a^4 - 192a^3b + 432a^2b^2 - 432ab^3 + 324b^4$
- $(x - 2y + 3z)^2 = \dots$
 - $3x^2 + y^2 + 4z^2 - 6xy + 12xz - 4yz$
 - $9x^2 + y^2 + 4z^2 - 6xy + 12xz - 4yz$
 - $x^2 + 4y^2 + 9z^2 - 4xy + 6xz - 12yz$
 - $9x^2 + y^2 + 4z^2 - 12xy + 24xz - 8yz$
 - $36x^2 + 4y^2 + 16z^2 - 12xy + 24xz - 8yz$
- $(x - 3y - 2z)^2 = \dots$
 - $x^2 + 9y^2 + 4z^2 - 6xy + 12xz - 4yz$
 - $x^2 + 9y^2 + 4z^2 - 6xy + 12xz - 4yz$
 - $x^2 + 9y^2 + 4z^2 - 6xy - 4xz + 12yz$
 - $9x^2 + y^2 + 4z^2 - 12xy + 24xz - 8yz$
 - $9x^2 + 4y^2 + 16z^2 - 12xy + 24xz - 8yz$
- $15x^2 + 12x =$
 - $x(x + 6)$
 - $3x(x + 6)$
 - $15(x + 1)$
 - $3x(5x + 4)$
 - $x(5x + 2)$
- $18a^5b^4 - 12a^2b^7 = \dots$
 - $4a^2b^4(3a^3 - 2b^3)$
 - $4ab^2(3a^4 - 2b^5)$
 - $6a^2b^4(3a^3 - 2b^3)$
 - $6a^3b^3(3a^2 - 2b^4)$
 - $6a^2b(3a - 2b^2)$
- $9x^2 - 25y^2 = \dots$
 - $(3x - y)(3x - 25y)$
 - $(3x - 5y)(3x + 5y)$
 - $(9x - y)(x - 25y)$
 - $(4x + 7y)(4x - 7y)$
 - $(4x + 49y)(4x - 49y)$
- $x^2 - 16 =$
 - $(x + 16)(x - 16)$
 - $(x + 2)(x - 8)$
 - $(x + 4)(x - 4)$
 - $(x - 4)(x - 4)$
 - $(x + 4)(x + 4)$

15. $16x^2 - 49y^2 = \dots$

- (A) $(3x - y)(3x - 25y)$
- (B) $(3x - 5y)(3x + 5y)$
- (C) $(9x - y)(x - 25y)$
- (D) $(4x + 7y)(4x - 7y)$
- (E) $(4x + 49y)(4x - 49y)$

16. $x^4 - 16 =$

- (A) $(x - 2)(x + 2)(x + 4)$
- (B) $(x - 2)(x + 2)(x - 4)$
- (C) $(x - 2)(x - 2)(x^2 + 4)$
- (D) $(x - 2)(x + 2)(x^2 + 4)$
- (E) $(x - 2)(x + 2)(x^2 - 4)$

17. $81m^4 - 625n^4 =$

- (A) $(3m - 5n)(3m + 5n)(9m^2 + 25n^2)$
- (B) $(3m - 5n)(3m - 5n)(9m^2 + 25n^2)$
- (C) $(3m - 5n)(3m + 5n)(9m^2 - 25n^2)$
- (D) $(3m - 5n)(3m + 5n)(9m + 25n)$
- (E) $(3m - 5n)(3m - 5n)(9m + 25n)$

18. $x^2 - 6x + 8 =$

- (A) $(x - 2)(x - 4)$
- (B) $(x + 2)(x + 4)$
- (C) $(x + 2)(x - 4)$
- (D) $(x + 4)(x - 4)$
- (E) $(x - 4)(x - 4)$

19. $x^2 + 7x + 12 =$

- (A) $(x - 2)(x + 6)$
- (B) $(x - 2)(x - 6)$
- (C) $(x - 3)(x + 4)$
- (D) $(x - 3)(x - 4)$
- (E) $(x + 3)(x + 4)$

20. $x^2 + 2x - 24 =$

- (A) $(x + 4)(x - 6)$
- (B) $(x - 4)(x - 6)$
- (C) $(x - 4)(x + 6)$
- (D) $(x + 12)(x - 2)$
- (E) $(x + 4)(x - 6)$

21. $2x^2 - 5x + 2 = \dots$

- (A) $(2x - 1)(x - 2)$
- (B) $(2x + 1)(x + 1)$
- (C) $(2x - 1)(x + 1)$
- (D) $(3x - 1)(x - 2)$
- (E) $(3x - 2)(x - 1)$

22. $3x^2 - 7x + 2 = \dots$

- (A) $(2x - 1)(x - 2)$
- (B) $(2x + 1)(x + 1)$
- (C) $(2x - 1)(x + 1)$
- (D) $(3x - 1)(x - 2)$
- (E) $(3x - 2)(x - 1)$

23. $\frac{3}{a} + \frac{5}{ab} =$

- (A) $\frac{3b + 5a}{ab}$
- (B) $\frac{3a - 5}{ab}$
- (C) $\frac{a + 5}{ab}$
- (D) $\frac{3b + 5}{ab}$
- (E) $\frac{3b - 5}{ab}$

24. $\frac{2}{x - 4} - \frac{3}{(x - 4)^2} = \dots$

- (A) $\frac{2x - 11}{(x - 4)^2}$
- (B) $\frac{2x + 4}{(x - 4)^2}$
- (C) $\frac{3x - 2}{(x - 4)^2}$
- (D) $\frac{3x + 15}{(x - 4)^2}$
- (E) $\frac{4x - 19}{(x - 4)^2}$

25. $\frac{4}{x^2 - 9} + \frac{3}{x + 3} =$

- (A) $\frac{3x - 5}{x^2 - 9}$
- (B) $\frac{3x + 5}{x^2 - 9}$
- (C) $\frac{3x + 12}{x^2 - 9}$
- (D) $\frac{4x + 15}{x^2 - 9}$
- (E) $\frac{4x - 9}{x^2 - 9}$

26. $\frac{24x}{x^2 - 6x + 8} - \frac{3}{x - 2} = \dots$

- (A) $\frac{21x + 3}{x^2 - 6x + 8}$
- (B) $\frac{27 + 12}{x^2 - 6x + 8}$
- (C) $\frac{5x + 3}{x^2 - 6x + 8}$
- (D) $\frac{5x + 17}{x^2 - 6x + 8}$
- (E) $\frac{21x + 12}{x^2 - 6x + 8}$

BAB 14

INTEGRAL

14.1 PENGERTIAN

Perhatikan pernyataan berikut :

$$F_1(x) = x^2 + 5x - 6 \text{ maka } F_1'(x) = 2x + 5$$

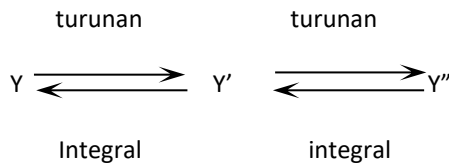
$$F_2(x) = x^2 + 5x + 12 \text{ maka } F_2'(x) = 2x + 5$$

$$F_3(x) = x^2 + 5x - \frac{3}{5} \text{ maka } F_3'(x) = 2x + 5$$

Pada fungsi-fungsi yang berbeda konstanta diperoleh bentuk turunan / derivatif yang sama.

Operasi dari $F(x)$ menjadi $F'(x)$ merupakan operasi turunan.

Sedangkan untuk operasi sebaliknya dari $F'(x)$ ke $F(x)$ disebut operasi INTEGRAL (ANTI TURUNAN).



Secara umum :

Jika $y' = \frac{dy}{dx}$ atau $dy = y' dx$ maka $\int dy = y = \int y' dx$

Dari 3 pernyataan di atas, nampak bahwa : operasi integral “ \int ” dari $F'(x)$ ke $F(x)$ yang berbeda adalah nilai konstanta akhir (-6, 12, - $\frac{3}{5}$).

Untuk $y = F(x) + C$ maka $y' = F'(x)$ dan dapat dituliskan

$$\int F'(x) dx = F(x) + c$$

14.2 RUMUS DASAR

1. Integral bentuk aljabar :

$$\int ax^n dx = \frac{a}{n+1} x^{n+1} + c \text{ (dengan } n \neq -1)$$

$$\int \frac{1}{x} dx = \int x^{-1} dx = \ln x + c$$

Contoh :

$$1. \int 12x^3 dx = 12 \cdot \frac{1}{4} x^3 + c = 3x^3 + c$$

$$2. \int 6x\sqrt{x} dx = \int x^{\frac{3}{2}} dx = \frac{6}{\frac{5}{2}} x^{\frac{5}{2}} + c = \frac{12}{5} x^{\frac{5}{2}} + c$$

$$3. \int \frac{7}{x^4} dx = \int 7x^{-4} dx = \frac{7}{-3} x^{-3} + c$$

$$4. \int \frac{8}{x\sqrt{x}} dx = \int 8x^{-\frac{3}{2}} dx = \frac{8}{-\frac{1}{2}} x^{-\frac{1}{2}} + c$$

$$= -16 x^{-\frac{1}{2}} + c$$

14.3 MACAMNYA

Berdasarkan batas, pernyataan integral dibedakan atas :

□ **Integral tak tentu** : Integral tanpa di sertai batas integrasi, ditulis : $\int F'(x) dx = F(x) + C$

□ **Integral tertentu** : Integral disertai batas integrasi, ditulis :

$$\int_a^b F'(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

Contoh :

$$1. \int (8x - 3) dx = 4x^2 - 3x + c$$

$$2. \int_1^3 (4x + 5) dx = (2x^2 + 5x) \Big|_1^3 = [2(9) + 5(3)] - [2(1) + 5(1)] = 26$$

3. Gradien garis singgung kurva $y = f(x)$ dinyatakan dengan $\frac{dy}{dx} = 2x + 5$ dan kuva tersebut melalui titik $P(1, 9)$. Tentukan persamaan $f(x)$!

Penyelesaian :

$$Y = \int y' dx = \int (2x + 5) dx$$

$$Y = x^2 + 5x + c, \text{ melalui } (1, 9), \text{ maka}$$

$$9 = 1 + 5 + c, \text{ sehingga } c = 3$$

$$\text{Jadi persamaan : } f(x) = x^2 + 5x + 3$$

4. Bila $F''(x) = 6x + 8$ menyatakan turunan kedua dari $F(x)$ dengan $F(1) = 11$ dan $F(2) = 39$ maka $F(-1) = \dots$

Penyelesaian :

$$F''(x) = 6x + 8$$

$$F'(x) = \int (6x+8) dx = 3x^2 + 8x + c$$

$$F(x) = \int (3x^2 + 8x + c) dx = x^3 + 4x^2 + cx + d$$

$$F(1) = 1 + 4 + c + d = 11 \dots (1)$$

$$F(2) = 8 + 16 + 2c + d = 39 \dots (2)$$

Eliminasi persamaan (1) dan (2) didapat $c = 9, d = -3$

$$F(x) = x^3 + 4x^2 + 9x - 3 \text{ dan } F(-1) = -9$$

14.4 SIFAT

$$1. \int \{ f(x) \pm g(x) \} dx = \int f(x) dx \pm \int g(x) dx$$

$$2. \int k f(x) dx = k \int f(x) dx$$

$$3. \int_a^b F(x) dx = - \int_b^a F(x) dx$$

$$4. \int_a^b F(x) dx + \int_b^c F(x) dx = \int_a^c F(x) dx$$

Contoh :

$$\begin{aligned}
 1. \int (6x^4 - 8x^3 + 5x - 9) dx &= \int 6x^4 dx - \int 8x^3 dx + \int 5x dx - \int 9 dx \\
 &= \frac{6}{5} x^5 - 2x^4 + \frac{5}{2} x^2 - 9x + c
 \end{aligned}$$

$$\begin{aligned}
 2. \int 7\sqrt[3]{x^4} dx &= 7 \int \sqrt[3]{x^4} dx \\
 &= 7 \int x^{\frac{4}{3}} dx \\
 &= 7 \cdot \frac{1}{\frac{7}{3}} x^{\frac{7}{3}} + c \\
 &= 3 x^{\frac{7}{3}} + c
 \end{aligned}$$

$$3. \int_1^3 (2x+7) dx = - \int_3^1 (2x+7) dx$$

$$4. \int_1^3 (2x+7) dx + \int_3^7 (2x+7) dx = \int_1^7 (2x+7) dx$$

14.5 TEKNIK PENGINTEGRALAN

A CARA BIASA

Diarahkan pada integral penjumlahan (+/-) 2 fungsi/lebih.

Contoh :

$$\begin{aligned}
 1. \int (12x^2 - 5\sqrt[3]{x^2} + \frac{3}{x^3} + \frac{9}{x\sqrt{x}}) dx &= \int (12x^2 - 5x^{\frac{2}{3}} + 3x^{-3} + 9x^{-\frac{3}{2}}) dx \\
 &= 4x^3 - 3x^{\frac{7}{3}} - \frac{3}{2}x^{-2} - 18x^{-\frac{1}{2}} + c
 \end{aligned}$$

$$\begin{aligned}
 2. \int 2x(3x-1) dx &= \int (6x^2 - 2x) dx \\
 &= 2x^3 - x^2 + c
 \end{aligned}$$

$$\begin{aligned}
 3. \int (x+3)(2x-5) dx &= \int (2x^2 + x - 15) dx \\
 &= \frac{2}{3}x^3 + \frac{1}{2}x^2 - 15x + c
 \end{aligned}$$

$$\begin{aligned}
 4. \int \left(\frac{x^2 + 5x + 3}{x\sqrt{x}} \right) dx &= \int \left(\frac{x^2}{x^{3/2}} + \frac{5x}{x^{3/2}} + \frac{3}{x^{3/2}} \right) dx \\
 &= \int \left(x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} + 3x^{-\frac{3}{2}} \right) dx \\
 &= \frac{2}{3}x^{\frac{3}{2}} + 10x^{\frac{1}{2}} - 6x^{-\frac{1}{2}} + C
 \end{aligned}$$

B. CARA SUBSTITUSI

B1. Bentuk LINIER

$$\int (ax+b)^n dx = \frac{1}{a} \cdot \frac{1}{n+1} \cdot (ax+b)^{n+1} + c$$

Contoh :

$$\begin{aligned}
 1. \int (3x+4)^6 dx &= \frac{1}{3} \cdot \frac{1}{7} (3x+4)^7 + c \\
 &= \frac{1}{21} (3x+4)^7 + c
 \end{aligned}$$

$$\begin{aligned}
 2. \int \sqrt{5x+3} dx &= \int (5x+3)^{1/2} dx \\
 &= \frac{1}{5} \cdot \frac{2}{3} (5x+3)^{3/2} + c \\
 &= \frac{2}{15} (5x+3)^{3/2} + c
 \end{aligned}$$

$$3. \int \frac{dx}{3x+5} = \int (3x+5)^{-1} dx = \frac{1}{3} \ln(3x+5) + c$$

$$\begin{aligned}
 4. \int \frac{6x+7}{3x-2} dx &= \int \left(2 + \frac{11}{3x-2} \right) dx \\
 &= \int \left(2 + 11(3x-2)^{-1} \right) dx \\
 &= 2x + \frac{11}{3} \ln(3x-2) + c
 \end{aligned}$$

B2. SUBSTITUSI : Bentuk Komposisi Fungsi

Bentuk umumnya : $\int F[g(x)] \cdot g'(x) dx$

Cara I :

Misal $u = g(x)$ dan $du = g'(x) dx$

didapat : $\int F(u) du$

Cara II :

dengan menggantikan $dx = \frac{dg(x)}{g'(x)}$ didapat :

$$\int F[g(x)] \cdot g'(x) dx = \int F[g(x)] \cdot g'(x) \cdot \frac{dg(x)}{g'(x)}$$

Contoh :

$$1. \int 4x(x^2+9)^5 dx = \dots$$

Misal : $u = x^2 + 9$ dan $du = 2x dx$, didapat :

$$\begin{aligned}
 \int 2(x^2+9)^5 2x dx &= \int 2(u)^5 du \\
 &= \frac{1}{3} u^6 + c \\
 &= \frac{1}{3} (x^2+9)^6 + c
 \end{aligned}$$

Atau :

$$\begin{aligned}
 \int 4x(x^2+9)^5 dx &= \int 4x(x^2+9)^5 \frac{d(x^2+9)}{2x} \\
 &= \int 2(x^2+9)^5 d(x^2+9) \\
 &= \frac{1}{3} (x^2+9)^6 + c
 \end{aligned}$$

$$2. \int \frac{x^2 dx}{\sqrt{x^3-8}} = \dots$$

Misal : $u = x^3 - 8$ dan $du = 3x^2 dx$ atau $x^2 dx = \frac{1}{3} du$

didapat :

$$\begin{aligned} \int (x^3-8)^{-\frac{1}{2}} x^2 dx &= \int (u)^{-\frac{1}{2}} \frac{1}{3} du \\ &= \frac{1}{3} \cdot 2 u^{1/2} + c \\ &= \frac{2}{3} \sqrt{x^3-8} + c \end{aligned}$$

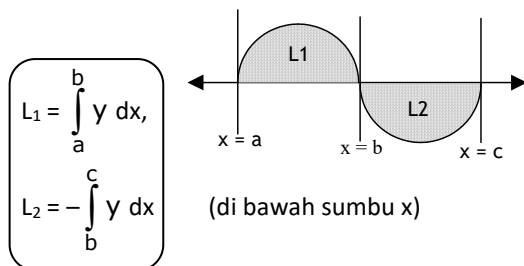
Atau :

$$\begin{aligned} \int (x^3-8)^{-\frac{1}{2}} x^2 dx &= \int (x^3-8)^{-\frac{1}{2}} x^2 \frac{d(x^3-8)}{3x^2} \\ &= \frac{1}{3} \int (x^3-8)^{-\frac{1}{2}} d(x^3-8) \\ &= \frac{1}{3} \cdot 2 \cdot (x^3-8)^{1/2} + c \\ &= \frac{2}{3} \sqrt{x^3-8} + c \end{aligned}$$

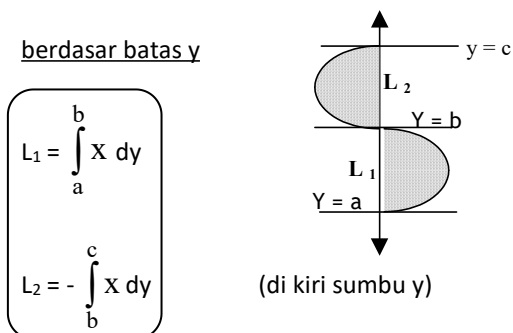
14.6 PENERAPAN INTEGRAL

A. Menghitung luas daerah

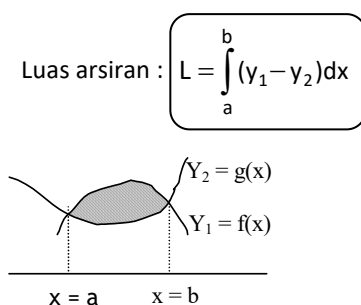
berdasar batas x



berdasar batas y

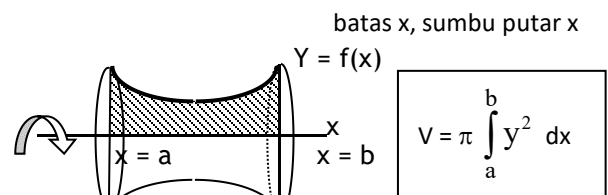


luas daerah diantara 2 kurva

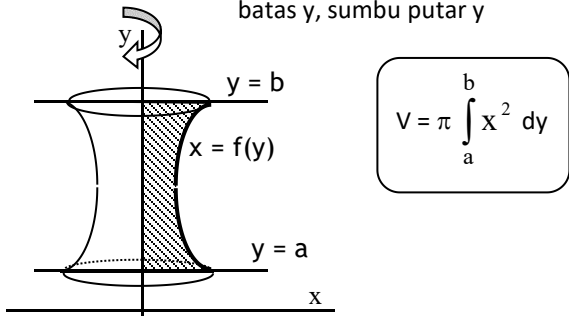


B. Menghitung volume benda putar

Kurva $y = f(x)$ diputar 360° terhadap sumbu x



Kurva $x = f(y)$ diputar 360° thd sb. y
batas y, sumbu putar y



contoh :

1. Hitunglah luas daerah yang dibatasi oleh parabola $y = x^2$ sumbu x, $x = 1$ dan $x = 3$

Penyelesaian :

Dibuat sketsa kurva $y = x^2$

Luas arsiran :

$$L = \int_1^3 y \, dx$$

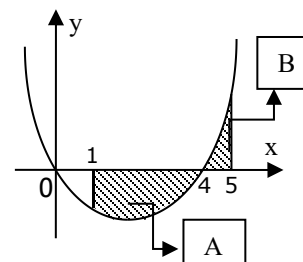
$$L = \int_1^3 x^2 \, dx = \frac{1}{3} x^3 \Big|_1^3 = \frac{1}{3} (27 - 1) = \frac{26}{3}$$

2. Hitunglah luas daerah yang dibatasi oleh parabola $y = x^2 - 4x$ sumbu x, $x = 1$ dan $x = 5$

Penyelesaian :

Dibuat sketsa kurva $y = x^2 - 4x$

Terdapat 2 bagian arsiran.



Daerah A (di bawah sumbu x) Daerah B (di atas sb. x)

Luas arsiran :

$$L_A = - \int_1^4 (x^2 - 4x) dx = \left(-\frac{1}{3}x^3 + 2x^2 \right) \Big|_1^4 = 9$$

$$L_B = \int_4^5 (x^2 - 4x) dx = \left(\frac{1}{3}x^3 - 2x^2 \right) \Big|_4^5 = \frac{7}{3}$$

$$\text{Luas total} = L_A + L_B = \frac{34}{3}$$

3. Hitunglah luas daerah yang dibatasi oleh kurva $y = x^2$ dan garis $y = 3x + 4$!

Penyelesaian :

sketsa kedua grafik dan dicari batas integral dari titik potong

$$\text{Titik potong : } y_p = y_g$$

$$x^2 = 3x + 4$$

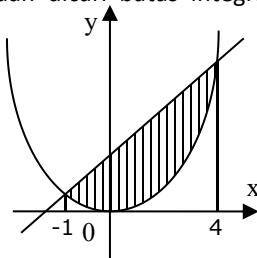
$$x^2 - 3x - 4 = 0$$

$$x = -1 \text{ atau } x = 4$$

Luas arsiran :

$$L_A = \int_{-1}^4 ((3x + 4) - x^2) dx$$

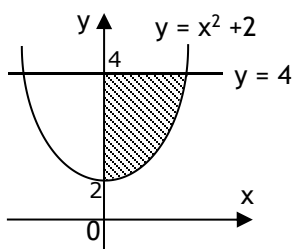
$$= \frac{125}{6}$$



4. Hitunglah luas daerah yang dibatasi oleh kurva $y = x^2 + 2$, sumbu y dan $y = 4$ di kuadran 1!

Penyelesaian :

Dibuat sketsa $y = x^2 + 2$. (dg batas y)



Luas arsiran :

$$L = \int_2^4 X dy$$

$$= \int_2^4 \sqrt{y - 2} dy = \frac{2}{3} (y - 2)^{3/2} \Big|_2^4$$

$$= \frac{4}{3} \sqrt{2}$$

5. Hitunglah volume, bila daerah yang dibatasi oleh Kurva $y = x^3$, sumbu x dan $x = 2$ diputar 360° terhadap sumbu x.

Penyelesaian :

Dibuat sketsa $y = x^3$.

Batas x, sumbu putar x

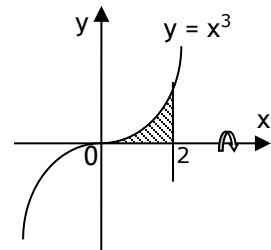
Volume :

$$V = \pi \int_0^2 y^2 dx$$

$$= \pi \int_0^2 x^6 dx$$

$$= \pi \frac{1}{7} x^7 \Big|_0^2$$

$$= \frac{128}{7} \pi$$



KAJI LATIH STANDAR 15

INTEGRAL

1. Ditetapkan $F'(x) = 9x^2 - 8x + 12$ dan $F(1) = 20$. $F'(x)$ adalah turunan dari $F(x)$, maka $F(x) = \dots$

- (A) $3x^3 - 4x^2 + 12x + 7$
- (B) $3x^3 - 4x^2 + 12x + 9$
- (C) $3x^3 + 4x^2 + 12x + 1$
- (D) $x^3 - 4x^2 + 12x + 9$
- (E) $x^3 - 4x^2 + 12x + 20$

2. Ditetapkan $F'(x) = \frac{6}{x^3} + 4x + 2$ dan $F(-1) = 2$. $F'(x)$ adalah turunan dari $F(x)$, maka $F(x) = \dots$

- (A) $-\frac{3}{x^2} + 2x^2 + 2x + 5$
- (B) $-\frac{3}{x^2} + 2x^2 + 2x + 4$
- (C) $-\frac{3}{x^2} + 2x^2 + 2x + 3$
- (D) $-\frac{3}{x^2} + 2x^2 + 2x - 3$
- (E) $-\frac{3}{x^2} + 2x^2 + 2x - 5$

3. $\int (2x^2 + 2) dx = \dots$

- (A) $\frac{1}{3}x^3 + 2x + C$
- (B) $2x^3 + 2x + C$
- (C) $\frac{1}{2}x^3 + 2x + C$
- (D) $\frac{2}{3}x^3 - 2x + C$
- (E) $\frac{2}{3}x^3 + 2x + C$

4. $\int (3x - 1)(x + 5) dx = \dots$

- (A) $\left(\frac{3}{2}x^2 - x\right)\left(\frac{1}{2}x^2 + 5x\right) + C$
- (B) $x^3 + 7x^2 + 5x + C$
- (C) $x^3 + 7x^2 - 5x + C$
- (D) $x^3 - 7x^2 + 5x + C$
- (E) $x^3 - 7x^2 - 5x + C$

5. $\int \left(\frac{24}{x^7} - \frac{42}{x^8}\right) dx = \dots$

- (A) $-\frac{4}{x^6} - \frac{6}{x^7} + C$
- (B) $\frac{4}{x^6} - \frac{6}{x^7} + C$
- (C) $-\frac{4}{x^6} + \frac{6}{x^7} + C$
- (D) $\frac{24}{7x^6} - \frac{42}{8x^7} + C$
- (E) $\frac{24}{7x^6} + \frac{42}{8x^7} + C$

6. $\int \left(8 \cdot \sqrt[3]{x} + \frac{1}{\sqrt{x}}\right) dx = \dots$

- (A) $\frac{1}{2}x + \sqrt[3]{x} + c$
- (B) $6x \cdot \sqrt[3]{x} + \frac{2}{\sqrt{x}} + C$
- (C) $6x \cdot \sqrt[3]{x} - \frac{2}{\sqrt{x}} + C$
- (D) $6x \cdot \sqrt[3]{x} + 2\sqrt{x} + C$
- (E) $8x \cdot \sqrt[3]{x} + 2\sqrt{x} + C$

7. $\int (5x + 6)^7 dx = \dots$

- (A) $\frac{5}{8}(5x + 6)^8 + C$
- (B) $\frac{1}{8}(5x + 6)^8 + C$
- (C) $\frac{1}{40}(5x + 6)^8 + C$
- (D) $7(5x + 6)^6 + C$
- (E) $35(5x + 6)^6 + C$

8. $\int_1^2 (3x^2 - 4x + 3) dx = \dots$

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

9. $\int_{-1}^2 \frac{8dx}{x^5} = \dots$

- (A) $\frac{5}{8}$
- (B) $\frac{63}{64}$
- (C) $-1\frac{1}{64}$
- (D) $1\frac{7}{8}$
- (E) $\frac{7}{8}$

10. $\int_{-1}^2 \frac{x^2 - 2}{x^4} dx = \dots$

- (A) $2\frac{1}{4}$
- (B) $1\frac{1}{4}$
- (C) $\frac{1}{4}$
- (D) $-\frac{3}{4}$
- (E) $-\frac{1}{12}$

11. $\int_1^3 (2x - 3)^3 dx = \dots$

- (A) 10
- (B) 20
- (C) 40
- (D) 80
- (E) 160

12. Jika $b > 0$, maka $\int_2^b (x + 5) dx = 16$, maka nilai $b = \dots$

- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) 7

13. Hasil $\int 2x\sqrt{2x^2 + 2} dx = \dots$

- (A) $x^2\sqrt{\frac{2}{3}x^3 + 2x} + C$
- (B) $\frac{4}{3}(x^4 + x^2)\sqrt{2x^2 + 2} + C$
- (C) $\frac{1}{3}(x^2 + 1)\sqrt{2x^2 + 2} + C$

(D) $\frac{2}{3}(x^2 + 1)\sqrt{2x^2 + 2} + C$

(E) $\frac{4}{3}(x^2 + 1)\sqrt{2x^2 + 2} + C$

14. Diketahui $f(x) = \frac{x^2 + 1}{\sqrt{x^3 + 3x - 4}}$ maka $\int f(x) dx = \dots$

(A) $\frac{1}{3}\sqrt{x^3 + 3x - 4} + C$

(B) $\frac{2}{3}\sqrt{x^3 + 3x - 4} + C$

(C) $\frac{1}{3}x\sqrt{x^3 + 3x - 4} + C$

(D) $-\frac{2}{3}\sqrt{x^3 + 3x - 4} + C$

(E) $\frac{2}{3}x^2\sqrt{x^3 + 3x - 4} + C$

15. Luas daerah yang dibatasi oleh garis $x + 4y = 8$, sumbu x dan sumbu y adalah.... Satuan luas

(A) $L = \int_0^8 (2 + \frac{1}{4}x) dx$

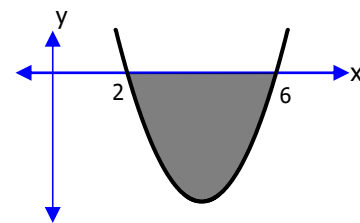
(B) $L = \int_0^8 (2 - \frac{1}{4}x) dx$

(C) $L = \int_0^4 (2 - \frac{1}{4}x) dx$

(D) $L = \int_4^8 (2 - \frac{1}{4}x) dx$

(E) $L = \int_0^8 (x + 4y - 8) dx$

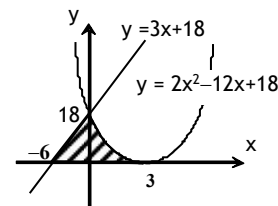
16. Luas daerah yang dibatasi oleh sumbu x dan parabola $y = 3x^2 - 24x + 36$ adalah



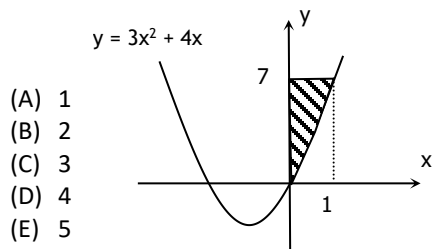
- (A) 24
- (B) 32
- (C) 48
- (D) 50
- (E) 64

17. Luas daerah yang diarsir pada gambar di bawah adalah

- (A) 18
- (B) 36
- (C) 54
- (D) 72
- (E) 80



18. Luas daerah yang diarsir pada gambar berikut adalah



19. Luas daerah yang dibatasi oleh sumbu x, sumbu y dan parabola $y = 3x^2 - 24x + 36$ adalah
(A) 28
(B) 30
(C) 32
(D) 36
(E) 42

20. Luas bidang yang dibatasi oleh $y = 2x^2$ dan $y = -x^2 + 6x$ adalah
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

